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6. Exercise on Convex Optimization

Problem 20: (Stationary points of a function)

For each value of the scalar β, find the set of all stationary points of the following function of the two variables x and y

$$f(x, y) = x^2 + y^2 + \beta xy + x + 2y.$$

Which of these stationary points are global minima?

Problem 21: (Local minima of a function)

Find all local minima of the function $f(x, y) = x^2/2 + x \cos y$ using optimality conditions.

Problem 22: (Steepest descent method with constant step size)

Describe the behavior of the steepest descent method with constant step size s for the function $f(x) = \|x\|^{2+\beta}$, where $\beta \geq 0$. For which values of s and x^0 does the method converge to $x^* = 0$? Relate your answer to the assumptions of Proposition 4.3.2.

Problem 23: (Applying gradient method on quadratic functions)

Suppose that f is quadratic and of the form $f(x) = x'Qx/2 - b'x$, where Q is positive definite and symmetric.

a) Show that the Lipschitz condition $\|\nabla f(x) - \nabla f(y)\| \leq L\|x - y\|$ is satisfied with L equal to the maximum eigenvalue of Q.

b) Consider the gradient method $x^{k+1} = x^k - sD\nabla f(x^k)$, where D is positive definite and symmetric. Show that the method converges to $x^* = Q^{-1}b$ for every starting point x^0 if and only if $s \in (0, 2/L)$, where L is the maximum eigenvalue of $D^{1/2}QD^{1/2}$.